MTH 1420, SPRING 2012
DR. GRAHAM-SQUIRE

SECTION 5.4: FUNDAMENTAL THEOREM OF CALCULUS

HW: 4, 5, 8, 14, 23

Practice: 3, 9, 13, 17, 19, 21

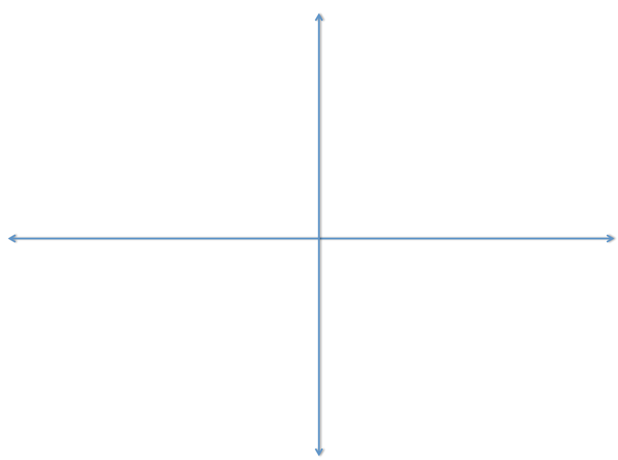
1. Introduction

As we have seen in the Evaluation Theorem of section 5.3 and Lab 2, the operations of differentiation and integration are inverses. Thus we can use anti differentiation to calculate integrals, and if we integrate and then differentiate we end up where we started. The lab gives an intuitive idea of the fundamental theorem, in this section we give precise statements of the theorem as well as investigate more complicated problems.

2. Examples

We begin with two quick examples to remind us about area functions and their derivatives:

Example 1. If $f(t) =$



and $g(x) = \int_0^x f(t)dt$, calculate $g(1) =$ ______, $g(2) =$ ______, $g(3) =$ ______, $g(4) =$ ______, $g(6) =$ ______

Exercise 2. Let $g(x) = \int_1^x (1 + \sqrt{t})dt$. Calculate $g(x)$ and $g'(x)$.

Remark. Notice that in the exercise above, you would get the same result for $g'(x)$ no matter what the lower limit of integration for $g(x)$ is. Thus although the function g might change if you adjust the lower limit of integration, the rate of change of g will be no different.

3. The Fundamental Theorem of Calculus

Theorem 3.1. *(Fundamental Theorem of Calculus)*

(1) *If f is continuous on $[a, b]$, and $g(x) = \int_a^x f(t)dt$ (for some x between a and b), then $g(x)$ is an antiderivative of f. Notationally, $g'(x) = f(x)$ for all $a < x < b$, and we can summarize it more succinctly as*

$$\frac{d}{dx}\int_a^x f(t)dt = f(x).$$

(2) $\int_a^b f(x)dx = F(b) - F(a)$, *where F is any antiderivative of F. (This is the same as the Evaluation Theorem from section 5.3)*

Proof. Proof of part 1 of the theorem: □

Exercise 3. Let $h(x) = \int_3^x \frac{\cos t}{t} dt$. Use the Fundamental Theorem to find $h'(x)$.

4. More tricky FTC questions

Example 4. Consider a question very similar to the exercise above, but with a different upper limit of integration. Let $h(x) = \int_3^{\sqrt{x}} \frac{\cos t}{t} dt$. Find $h'(x)$.

Example 5. Another method for solving the example above is:

Exercise 6. Find the derivative of the function:

$$\int_0^{\tan x} \sqrt{t + \sqrt{t}}\, dt$$

Exercise 7. Find a function f such that $f(2) = 0$ and $f'(x) = \dfrac{\sin x}{x}$. Hint: Use the fundamental theorem of calculus to find an antiderivative for f', then choose the correct lower limit of integration so that $f(2) = 0$.